Thus for any f_i internal to a series of f (i.e., neither the first nor the last element) the noise contribution has two parts, each contained with opposite sign in an adjacent element of f. In this sense the noise may be said to be perfectly correlated.

In order to be able to include this noise covariance matrix in a filter, it must be inverted, and we seek this inverse in a convenient form. The determinant D_N of the preceding matrix U of dimension N satisfies a recurrence relation

$$D_N = D_{N-1} - \alpha^2 D_{N-2} \tag{5}$$

Given the determinant, the inverse can be found by

$$(U^{-1})_{ij} = D_i D_{N-j}(-\alpha)^{j-1}/D_N; \qquad j \geq i \tag{6}$$

This inverse, although possibly useful for numerical (digital computer) computations does not lend itself to further analytic study. Fortunately, for the important case $\alpha = -\frac{1}{2}$, the inverse can be found in a closed form

$$[U^{-1}(-\tfrac{1}{2})]_{ij} = [2/(N+1)](N+1-j)i; \qquad j \geq i \tag{7}$$

which is convenient for analysis.

In order to gain a feeling for the effect of perfect noise correlation on the estimation of parameters, we consider two special cases. In each a single parameter x is estimated with both the perfectly-correlated-noise and white-noise assumptions. The observed quantity is the Doppler frequency $f_i = f(x, t_i)$ and the cases differ only in the form of the sensitivity $\mathbf{A}$ which is defined by

$$\mathbf{A}_i = \partial f_i/\partial x \tag{8}$$

Case I: The Sensitivity is Constant, $\mathbf{A}_i = A$

If the data were uncorrelated [i.e., the noise statistics of Eq. (3b) with $\alpha = 0$] then the parameter variance would be

$$P_{IW} = 2Q/\tau^2 A^2 N \tag{9}$$

and the data would be equally weighted in the parameter estimation procedure. With the perfectly correlated noise ($\alpha = -\frac{1}{2}$) the variance of the parameter is

$$P_{IC} = [\mathbf{A}^\dagger R^{-1}\mathbf{A}]^{-1} \tag{10a}$$

$$= \frac{2Q}{\tau^2 A^2}\left[\sum_{ij}(U^{-1})_{ij}\right]^{-1} \tag{10b}$$

$$= 12Q/\tau^2 A^2 N(N+1)(N+2) \tag{10c}$$

Similarly, the estimate $\hat{x}$ of the parameter x is given by

$$\hat{x} = x_0 + P_{IC}\,\mathbf{A}^\dagger R^{-1} u \tag{11a}$$

$$= x_0 + (1/A)W^\dagger u \tag{11b}$$

where u is the prefit residual vector

$$u_i = f_i\text{ (observed)} - f(x_0, t_i) \tag{12}$$

and

$$W_k = 6k(N+1-k)/N(N+1)(N+2) \tag{13}$$

acts like a weighting vector.[1]

The information content ratio G_I follows from Eqs. (9) and (10c)

$$G_I = P_{IW}/P_{IC} = (N+1)(N+2)/6 \tag{14}$$

Case II: The Sensitivity is Periodic, $\mathbf{A}_i = A\sin(2\pi t_i/M\tau + \phi)$

We assume that $1 \ll M \ll N$, replace all sums by integrals, and average over the phase ϕ as well as all effects due to the commensurability of M and N. Again starting with the white-noise assumption the parameter variance is

$$P_{IIW} = 4Q/\tau^2 A^2 N \tag{15}$$

With the perfectly-correlated-noise assumption the parameter variance is

$$P_{IIC} = 8Q\pi^2/\tau^2 A^2 N M^2 \tag{16}$$

yielding an information content ratio

$$G_{II} = M^2/2\pi^2 \tag{17}$$

Further Considerations

Having shown that the perfect noise correlation has a drastic effect on the information content of a set of data, we examine the result of processing such data with the white-noise assumption. This estimate $\hat{x}$ of the parameter x has covariance P^*

$$P^* = \langle(\hat{x}-x)(\hat{x}-x)^\dagger\rangle \tag{18}$$

$$\hat{x} - x = [\mathbf{A}^\dagger R^{-1}\mathbf{A}]^{-1}\mathbf{A}^\dagger R^{-1}u^* \tag{19}$$

$$u_i^* = \frac{v_i - v_{i-1}}{\tau} \tag{20}$$

$$P^* = [\mathbf{A}^\dagger\mathbf{A}]^{-1}\mathbf{A}^\dagger U(-\tfrac{1}{2})\mathbf{A}[\mathbf{A}^\dagger\mathbf{A}]^{-1}\langle v^2\rangle 2/\tau^2 \tag{21}$$

where R is the nominal (diagonal) noise covariance. Applying this expression to the preceding two cases we find

$$P_I^* = (1/N)P_{IW} = [(N+1)(N+2)/6N]P_{IC} \tag{22}$$

$$P_{II}^* = P_{IIC} \tag{23}$$

Thus in case I the white-noise assumption used with perfectly-correlated data causes a substantial loss of information, while in case II there is no loss of information. This latter result does not generally hold when the Fourier transform of the sensitivity contains more than a single spike.

It has been shown that the amount of information which can be extracted from a set of data may be larger if the associated noise is perfectly correlated than if it is white, and that the use of the white-noise assumption in the filter may result in the loss of some of this information. By simple extension it is to be expected that similar conclusions can be drawn for other types of non-white noise, and that in some cases the inclusion of a correct noise covariance may produce a spectacular improvement in the performance of a filter.

The form of the weighting vector suggests a new data type z which are half as numerous as the f and have a diagonal covariance matrix $\bar{R}$

$$z_k = \sum_{i=k}^{n+1-k} f_i = \frac{n_{N+1-k} - n_{k-1}}{(N+2-2k)\tau} \qquad \begin{array}{ll} k = 1,2,\ldots,N/2; & N\text{ even} \\ k = 1,2,\ldots,(N+1)/2; & N\text{ odd}\end{array}$$

$$\bar{R}_{kj} = \delta_{kj}2Q^2/\tau^2(N+2-2k)^2$$

Symmetries of the Boundary-Layer Equations under Groups of Linear Transformations

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The Equations and Transformation Groups

The boundary-layer equations are[1]

$$uu_x + vu_y = UU_x + \varepsilon^2 u_{yy} \tag{1}$$

$$u_x + v_y = 0 \tag{2}$$

with the boundary conditions

$$u = v = 0 \quad \text{at} \quad y = 0 \tag{3}$$

and

$$u(x, y) \to U(x) \quad \text{as} \quad y \to \infty \tag{4}$$

together with an initial profile $u(x_0, y)$.

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The best known transformation is the "Prandtl stretching" defined, for small ε, by

$$x \to x: \quad y \to \varepsilon y: \quad u \to u: \quad v \to \varepsilon v: \quad \varepsilon^2 \cong 1/R \neq 0 \qquad (5)$$

The transformation preserves the form of the equations and the coefficient of u_{yy} in Eq. (1) becomes unity. This "stretching" has been thoroughly discussed elsewhere.[1]

Birkhoff[2] suggests the following one parameter group:

$$x \to \beta^2 x: \quad y \to \beta y: \quad u \to u: \quad v \to \beta^{-1} v \qquad (6)$$

Under this transformation the form of Eqs. (1) and (2) is preserved with all terms having a multiplying factor of $1/\beta^2$. Thus for $\beta \neq 0$ or ∞ the group defined by Eq. (6) leaves the boundary-layer equations invariant.

Finally we discuss the general four-parameter group[2] described by

$$x \to \alpha x: \quad y \to \beta y: \quad u \to \gamma u: \quad v \to \delta v \qquad (7)$$

Birkhoff discusses the application to flow past an infinite wedge for which $U = cx^m$. This analysis reduces the boundary-layer equations to the usual Falkner-Skan ordinary differential equation. We are interested in generalized flow around a cylinder of the form

$$U = m \sin x \qquad (8)$$

We wish to know for what subgroups of (7) the boundary-layer equations are invariant with respect to variations in m. We also wish to preserve the form of U. Transforming the equations by means of (7) and equating the coefficients of different terms in each equation it is found that

$$\alpha = 1: \quad \beta = 1/m^{1/2}: \quad \gamma = m: \quad \delta = m^{1/2} \qquad (9)$$

where $m \neq 0$, ∞. Thus the required transformation is a one-parameter subgroup of (7) uniquely defined by

$$x \to x: \quad y \to y/m^{1/2}: \quad u \to mu: \quad v \to m^{1/2} v \qquad (10)$$

It is worth noting that $u_y \to m(m)^{1/2} u_y$. These results have the following application. It is often useful to be able to compare the boundary-layer solutions of different workers who have used different values of m for the external flow. The transformation (10) gives the scaling that must be used on the coordinates and velocities to compare solutions that have different m values. Typical values are $m = 1$ for Terrill's solution (Ref. 3) and $m = 2$ for Schonauer's solution (Ref. 4). The velocity profiles, at $x = 1.0$, obtained from Refs. 3 and 4 are compared by plotting u/U vs $y/m^{1/2}$ on Fig. 1. It is seen that the two solutions fall on the same curve, confirming the prediction of Eq. (10).

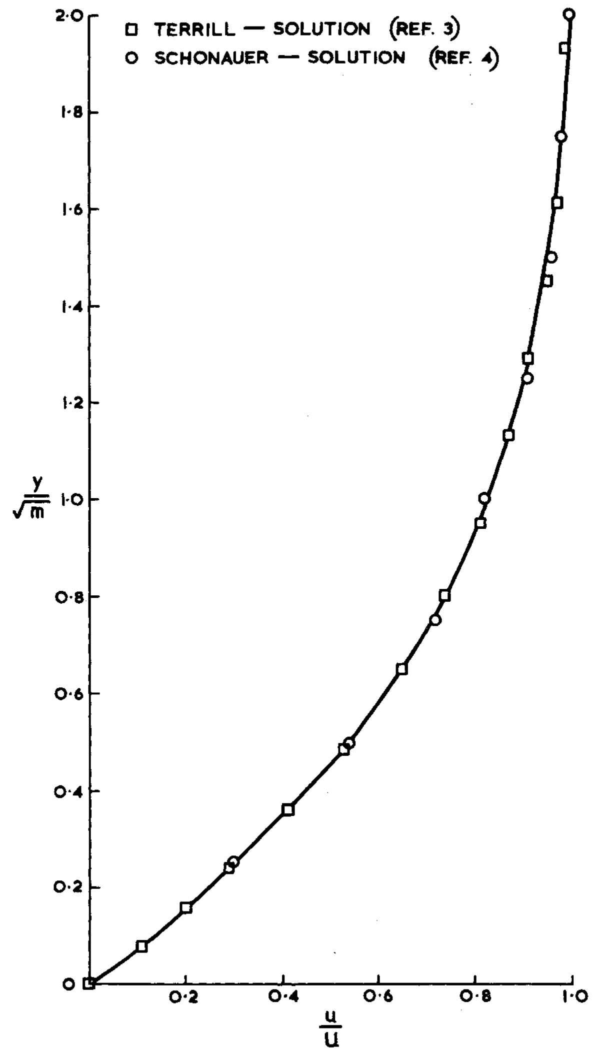

Fig. 1 Boundary-layer velocity profile for potential flow past a cylinder. Terrill and Schonauer solutions at $x = 1.0$.

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Buckling and Vibration Analysis for Stiffened Orthotropic Shells of Revolution

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Introduction

THE multistep integration method has proven to be a usefı and versatile approach in the analysis of rotationall symmetric shells. The present Note provides a numerical metho using the multistep integration approach, in which the bucklir and vibration analyses are formulated as a succession of line eigenvalue problems. The new method has some definite adva tages over the previous ones.[1] An estimate of the eigenvect is not required, and once an eigenvalue has been converg good estimates for other eigenvalues are automatically availabl This is accomplished through the use of an in-core Household scheme for solution of the eigenvalue problem. Furthermo since the method uses an eigenvalue solution, the possibility missing modes is eliminated.

Formulation

The shell is considered as being made up of a series of se ments, having prescribed lengths, which are connected alo

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